

CSL Collapse Model And Spontaneous Radiation: An Update

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Abstract

A brief review is given of the Continuous Spontaneous Localization (CSL) model in which a classical field interacts with quantized particles to cause dynamical wavefunction collapse. One of the model's predictions is that particles "spontaneously" gain energy at a slow rate. When applied to the excitation of a nucleon in a Ge nucleus, it is shown how a limit on the relative collapse rates of neutron and proton could be obtained, and a rough estimate is made from data. When applied to the spontaneous excitation of 1s electrons in Ge, by a more detailed analysis of more accurate data than previously given, an updated limit is obtained on the relative collapse rates of the electron and proton, suggesting that the coupling of the field to electrons and nucleons is mass proportional.

1. Introduction

It is appropriate to discuss comparison of experiment to a theory with fundamental pretensions in a volume dedicated to Dan Greenberger whose own theoretical work of a fundamental nature has seldom been far from testability.

In standard quantum theory (SQT) the statevector evolves in two ways. One evolution proceeds smoothly via Schrodinger's equation. The other is the abrupt (and ill defined) "collapse" of the statevector. This is the replacement of a statevector equal to a sum of vectors (each describing a different outcome of an "experiment") by one vector in the sum. One might guess that this dual evolution is indicative of a fundamental deficiency in present day physics. In the hope of finding new physics, one may begin by trying to modify Schrodinger's equation so that the statevector undergoes only a smooth evolution, giving both the usual quantum behavior and the collapse behavior.

This program, begun three decades ago,^{1,2} received a crucial impetus one decade ago from the work of Ghirardi, Rimini and Weber (GRW)³ and has evolved into the Continuous Spontaneous Localization (CSL) model.⁴ At present, this is the only fully developed nonrelativistic collapse model, with definite predictions applicable to any nonrelativistic experimental situation.

In the CSL model, a term which depends upon a randomly fluctuating field $w(\mathbf{x}, t)$ is added to Schrodinger's equation. (The physical nature of this field is unspecified, but metric fluctuations,⁵ possibly with a tachyonic spectrum,⁶ have been suggested.) The probabilistic behavior of Nature is explained as due to our lack of control of the field w . When an experiment is under way, the particles in the system+apparatus interact with the particular sample field w that is present, causing a rapid evolution of the statevector to one of the alternative outcomes of the experiment. A different sample field leads to a different outcome. CSL also specifies the probability that a particular sample field $w(\mathbf{x}, t)$ actually occurs. When all possible fields are taken into account, together with their probabilities, the result is that each outcome occurs with (essentially — see next paragraph) the probability predicted by SQT. Thus two relations, the Modified Schrodinger Equation and the Probability Rule constitute the CSL model.

As with any modification of SQT, one expects—and hopes—for certain specially designed experiments where SQT and CSL lead to different predictions, making tests possible. For example, one such test, presently not practicable, is a two slit interference experiment with a sufficiently large bound state object. Once the two wavepackets for the object leave the slits, SQT says that their amplitudes will never change so that interference is possible at any time. CSL says that the amplitudes will fluctuate and, after a long enough wait, eventually one of the packets will become negligible in amplitude, giving no interference pattern. Such an interference experiment with e.g., 90Å diameter drops of mercury⁷ over a time interval of seconds could provide such a test. However, this is a difficult experiment. For example, it is hard to prevent the two packets from being put into different angular momentum eigenstates by interaction with the environment, and then they would not interfere for this reason.⁸

The tests that are most practicable at present stem from the consequence of CSL that the collapse process imparts energy to particles. (One may think of this energy as provided by the field w .) The reason is that collapse entails the narrowing of wavefunctions. By the uncertainty principle, this leads to an increased momentum spread, and thus to an increased energy. Thus, any bound ground state of, e.g., atoms or nuclei, will be excited by the collapse part of the Schrodinger equation.^{3,4,9} The usual part of the Schrodinger equation will describe the radiation emitted as the system returns to the ground state. Also, a free charged particle is “shaken” by the field w , and so it will radiate¹⁰. Similarly, the quarks inside a proton should be excited, and the proton should radiate mesons.¹¹ Thus, a signature of CSL is that matter should emit “spontaneous radiation.” In section 3, various radiation rates are given.

It is interesting that, at this time, quite a number of low noise experiments are being undertaken to look for radiation appearing in an apparatus for a variety of reasons, e.g., because of collisions with purported Dark Matter. Some of these experiments are sensitive enough to provide useful constraints on the parameters of CSL.

Some experiments which look at radiation appearing in a slab of Germanium are described in section 4. In section 5 we show how data from one such experiment, “Rico Grande,” applied to the spontaneous excitation of a proton in a Ge nucleus, can provide a limit on the relative collapse rates of neutron and proton. Only a rough estimate is given because greater precision requires a more careful calculation of Ge nuclear dipole matrix elements than we are prepared to give here.

In section 6 we consider the spontaneous ionization rate of a 1s electron in a Ge atom. In a previous paper,¹² it was argued that the upper limit on this rate given by a single data point from the “TWIN” experiment¹³ suggested that the coupling of the field w to an electron or nucleon (here assumed to be the same for neutron and proton) is proportional to the particle’s mass, supporting previous proposals that there is a connection between gravity and collapse^{5,14,15}. Unfortunately it subsequently turned out that the data point was inaccurate (see section 4). However, the more complete analysis on “COSME” data presented here gives essentially the previous result.

Eventually, from such experiments, one may hope that the collapse rate parameter of CSL will either be constrained to be so small that the model will be ruled out—or that spontaneous radiation from collapse will actually be observed!

2. CSL

Underlying CSL are two mechanisms. One is the Gambler’s Ruin mechanism. This explains how the random process embodied in the noise w produces the probabilities of SQT for the collapsed states.¹⁶ The other is the GRW “hitting” mechanism. It allows collapse to occur rapidly, for macroscopic objects, to states which we see around us (localized objects), while microscopic objects are scarcely affected.³

Here is the Gambler’s Ruin analogy. Suppose, at the beginning of a game, gambler 1 (2) starts with $d_1(0)$ ($d_2(0)$) dollars, and $d_1(0) + d_2(0) = 100$. This is to be analogous to the initial statevector $|\psi(0)\rangle = a_1(0)|1\rangle + a_2(0)|2\rangle$, with the correspondence $d_i(0)/100 \rightarrow |a_i(0)|^2$. The gamblers toss a coin (analogous to the fluctuating field w) and, depending on the result, one gives a dollar to the other. As the game proceeds, the $d_i(t)$ fluctuate, just as do the squared amplitudes $|a_i(t)|^2$. One gambler finally wins all the money and the game stops, with e.g., gambler 2 winning with probability $d_2(0)/100$. Precisely analogously, collapse finally occurs, e.g., with $|\psi(t)\rangle \rightarrow 0|1\rangle + 1|2\rangle$ with probability $|a_2(0)|^2$. This is, of course, the probability

predicted by SQT of the outcome $|2\rangle$ if $|1\rangle$ and $|2\rangle$ represent two states of an apparatus.

The GRW model postulates a physical process which produces a sudden random change (“hit”) of a many-particle wavefunction: the wavefunction is multiplied by a gaussian function $\exp -(\mathbf{x}_n - \mathbf{z})^2/2a^2$, where \mathbf{x}_n is the position coordinate of the n th particle. The center of the gaussian, \mathbf{z} , is chosen according to a Probability Rule which depends in a certain way upon the wavefunction, making it most likely that \mathbf{z} is located where the wavefunction is largest. The effect of a hit is to narrow to width a the part of the wavefunction which depends upon the n th particle: GRW chose the mesoscopic length $a \approx 10^{-5}$ cm. A hit on one particle occurs at a slow rate λ : GRW chose $\lambda \approx 10^{-16}\text{sec}^{-1} \approx$ once in 300 million years. But, each particle is equally likely to be hit, so a hit on an N particle object occurs rapidly, on average in $1/\lambda N\text{sec}$. The wavefunction of an N particle object in a superposition of states, each describing the object in a different place, has the particles entangled in such a way that one hit on one such particle causes the wavefunction to collapse in $1/\lambda N\text{sec}$ to one of the states in the superposition. This explains how macroscopic objects are always observed as localized. (A defect of this model is that the (anti) symmetry of the wavefunction is destroyed by the hitting process.)

CSL may be thought of as embodying a continuous hitting process. A hit occurs every Δt sec, but the hit wavefunction is multiplied by Δt and added to the original wavefunction. Thus the wavefunction shape fluctuates gradually (not suddenly as in GRW’s model), and the gambler’s ruin dynamics is obtained. The (anti) symmetry of the wavefunction is properly preserved in CSL.

The CSL modified Schrodinger equation is

$$\frac{d|\psi, t >_w}{dt} = -iH|\psi, t >_w - \frac{1}{4\lambda} \int d\mathbf{x} [w(\mathbf{x}, t) - 2\lambda A(\mathbf{x})]^2 |\psi, t >_w \quad (2.1)$$

Given an arbitrary field $w(\mathbf{x}, t)$, one may solve (2.1) to find how the statevector evolves under its influence. Operator $A(\mathbf{x})$ is

$$A(\mathbf{x}) \equiv \sum_{\alpha} g_{\alpha} \frac{1}{(\pi a^2)^{\frac{3}{4}}} \int d\mathbf{z} N_{\alpha}(\mathbf{z}) e^{-\frac{(\mathbf{x} - \mathbf{z})^2}{2a^2}} \quad (2.1a)$$

The integral in (2.1a) is essentially the number of particles in a sphere of diameter a centered around \mathbf{x} . $N_{\alpha}(\mathbf{z})$ is the number density operator for particles of type α , e.g., electrons, protons and neutrons, and λg_{α}^2 is their one-particle collapse rate (see Eq. (2.5) below).

Thus, the parameters which characterize CSL are a , λ and the ratios of the g_{α} ’s, e.g., for ordinary matter, g_n/g_p and g_e/g_p where the subscripts p , n , and e refer to the proton, neutron and electron. With no loss of generality we may take $g_p = 1$, so λ is an individual proton’s collapse rate.

CSL requires a second equation, giving the probability density $P(w)$ functional that the field $w(\mathbf{x}, t)$ occurs:

$$P(w) = {}_w\langle \psi, t | \psi, t >_w \quad (2.2)$$

Because the evolution (2.1) is nonunitary, the norm of the statevector $|\psi, t >_w$ changes with time. Eq.(2.2) says that the most probable fields to occur are those which lead to statevectors of largest norm.

Eqs.(2.1) and (2.2) ensure that an initial statevector, which is in a superposition of states of different particle number density, evolves toward one of these states for each probable w —and that the ensemble of all evolutions is such that each final state occurs with (essentially) the SQT probability. However, we shall not discuss here how Eqs.(2.1), (2.2) lead to collapse for an individual statevector. Instead we shall go right to the appropriate object for discussing experimental predictions, the density matrix ρ . The evolution equation for the density matrix which follows from Eqs.(2.1), (2.2) can be shown to be ⁴

$$\begin{aligned} \frac{\partial \langle x | \rho(t) | x' \rangle}{\partial t} &= -i \langle x | [H, \rho(t)] | x' \rangle \\ &- \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^N g_{\alpha(j)} g_{\alpha(k)} [\Phi(\mathbf{x}_j - \mathbf{x}_k) + \Phi(\mathbf{x}'_j - \mathbf{x}'_k) - 2\Phi(\mathbf{x}_j - \mathbf{x}'_k)] \langle x | \rho(t) | x' \rangle \end{aligned} \quad (2.3)$$

where $|x\rangle = |\mathbf{x}_1, \mathbf{x}_2, \dots\rangle$ is the position eigenstate for all particles and

$$\Phi(\mathbf{z}) \equiv e^{-\frac{\mathbf{z}^2}{4a^2}} \quad (2.4)$$

As a simple example of how Eq. (2.3) works, set $H = 0$ so as to concentrate on the collapse dynamics alone, and consider a clump of particles of type α in a superposed state. That is, let the initial state be $a_1(0)|1\rangle + a_2(0)|2\rangle$, where $|1\rangle$ and $|2\rangle$ each describe N particles of type α in a localized state with dimensions $\ll a$, but with centers of mass of the two states at a distance $\gg a$ apart. Then $\Phi(\mathbf{x}_j - \mathbf{x}_k) \approx 1$ if $\mathbf{x}_j, \mathbf{x}_k$ are both located in region 1 (or both in 2) and $\Phi(\mathbf{x}_j - \mathbf{x}'_k) \approx 0$ if $\mathbf{x}_j, \mathbf{x}'_k$ are located in regions 1 and 2 respectively. Therefore, Eq.(2.3) yields

$$\frac{\partial \langle 1|\rho(t)|2\rangle}{\partial t} = -\frac{\lambda g_\alpha^2}{2} \sum_{j=1}^N \sum_{k=1}^N (1 + 1 - 2 \cdot 0) \langle 1|\rho(t)|2\rangle = -\lambda g_\alpha^2 N^2 \langle 1|\rho(t)|2\rangle \quad (2.5)$$

showing that the off-diagonal elements of ρ decay at the rate λN^2 . This illustrates how the collapse rate is large for a superposition of states describing a large number of particles in different locations.

3. Excitation Rate Predictions

As mentioned in section 1, a byproduct of the collapse process is that particles gain energy. It is easy to show, using Eq.(2.3), that the average total energy $\bar{H}(t) \equiv \text{Tr} H \rho(t)$ ($H = \sum_{i=1}^N \mathbf{p}_i^2 / 2M_i + V(\mathbf{x}_1 \dots \mathbf{x}_N)$) increases according to

$$\frac{d\bar{H}(t)}{dt} = \frac{3\lambda}{2} \sum_{j=1}^N g_{\alpha(j)}^2 \frac{\hbar^2}{2M_j a^2} \quad (3.1)$$

Assuming $g_\alpha = 1$ for all particles, and using the GRW values for λ and a , then 10^{24} nucleons gain $\approx .3$ eV/sec and 10^{24} electrons gain ≈ 600 eV/sec. This is quite small, corresponding to a temperature increase over the age of the universe of $\approx .001^\circ\text{K}$ and 2°K respectively. (The low particle density in the universe assures that the effect of this increased energy on the cosmic radiation bath is negligible). If we take 10^{24} electrons as roughly the number in a cc. of condensed matter, this corresponds to an energy increase of about 10^{-15} joules/sec, which is close to the experimentally detectable lower bound by present day bolometric measurements.¹⁷ This is much less sensitive than the experiment discussed here.

However, while (3.1) gives the *average* behavior, there are infrequent but large energy fluctuations.

The energy increase in Eq.(3.1) is the sum of increased internal energy and of increased center of mass energy $H_{cm} \equiv \mathbf{P}_{cm}^2 / 2M + V_{cm}(\mathbf{Q})$ ($\mathbf{P}_{cm} \equiv \sum_{j=1}^N \mathbf{p}_j$, $M \equiv \sum_{j=1}^N M_j$, $\mathbf{Q} \equiv \sum_{j=1}^N M_j \mathbf{x}_j / M$). From Eq.(2.3) we find

$$\begin{aligned} \frac{d\bar{H}_{cm}(t)}{dt} &= \frac{3\lambda\hbar^2}{4a^2 M} \sum_{j=1}^N \sum_{k=1}^N g_{\alpha(j)} g_{\alpha(k)} \text{Tr} \left\{ \left[1 - \frac{(\mathbf{x}_j - \mathbf{x}_k)^2}{6a^2} \right] e^{-\frac{(\mathbf{x}_j - \mathbf{x}_k)^2}{4a^2}} \rho(t) \right\} \\ &= \frac{3\lambda\hbar^2}{4a^2 M} \left[\sum_{j=1}^N g_{\alpha(j)} \right]^2 - o(a^{-4}) \end{aligned} \quad (3.1a, b)$$

where the first term in the expansion (3.1b) dominates if the system under consideration, like an atom or nucleus, has dimensions $\ll a$. The condition for the total energy increase to be completely due to the center of mass energy increase to order a^{-2} , i.e., for there to be no internal excitation to this order, is found by equating (3.1) to (3.1b):

$$0 = \sum_{j=1}^N g_{\alpha(j)}^2 / M_j - \left[\sum_{j=1}^N g_{\alpha(j)} \right]^2 / M = (2M)^{-1} \sum_{j=1}^N \sum_{k=1}^N \left[g_{\alpha(j)} \sqrt{M_k / M_j} - g_{\alpha(k)} \sqrt{M_j / M_k} \right]^2$$

i.e., if $g_{\alpha(j)} = CM_j$. Therefore, for such mass-proportionality of the coupling constants, the internal energy does not increase to order a^{-2} . Moreover, the leading term in the internal energy increase, proportional to a^{-4} , is compensated by an identical decrease in the center of mass energy since, by (3.1), the total energy increase vanishes to order a^{-4} and higher.

If the coupling constants are not mass-proportional, the internal excitation rate $\sim a^{-2}$ is found as follows. Consider a transition from a state $|\psi\rangle$ to a state $|\phi\rangle$, where $|\psi\rangle$ is an initial bound state, $|\phi\rangle$ is an orthogonal final state, $|\chi\rangle$ is an initial state of the center of mass and $|\chi'\rangle$ is an arbitrary final state of the center of mass. The probability per second of a transition from $|\psi\rangle$ to $|\phi\rangle$, regardless of the final center of mass state is $\dot{P} \equiv \sum_{|\chi'\rangle} \langle \chi' | \langle \phi | \dot{\rho}(0) | \phi \rangle | \chi' \rangle$, where $\rho(0) = |\chi\rangle \langle \psi | \langle \psi | \langle \chi |$. Expansion of Eq. (2.3) to first order in (dimension of system/ a)² yields¹¹

$$\dot{P}_1 = \frac{\lambda}{2a^2} \langle \phi | \mathbf{R} | \psi \rangle \cdot \langle \psi | \mathbf{R} | \phi \rangle \quad (3.2)$$

where $\mathbf{R} \equiv \sum_{j=1}^N g_{\alpha(j)} \mathbf{R}_j$, $\mathbf{R}_j \equiv \mathbf{x}_j - \mathbf{Q}$.

It is the predictions of Eq.(3.2) that we shall test in this paper. It is worth remarking that the matrix element in Eq.(3.2) involving a charged particle's \mathbf{R}_j is the same as that involved in describing an electric dipole transition between $|\psi\rangle$ and $|\phi\rangle$. Thus one can evaluate this contribution to (3.2) either by calculating the relevant matrix element or by expressing it in terms of measurable transition rates. Indeed, as we shall see in sections 5 and 6, using $\mathbf{R} \equiv 0$ if $g_\alpha \sim M_\alpha$, it is possible to express the matrix elements of one type of particles in terms of another type so one need only calculate or measure the matrix elements of the excited particle type to evaluate (3.2).

Incidentally, by summing Eq.(3.2) over all states $|\phi\rangle$ orthogonal to $|\psi\rangle$, we obtain \dot{P}_1^T , the total probability/sec for excitation of $|\psi\rangle$:

$$\dot{P}_1^T = \frac{\lambda}{2a^2} \langle \psi | [\mathbf{R} - \langle \psi | \mathbf{R} | \psi \rangle]^2 | \psi \rangle \quad (3.3)$$

Assume the GRW values for λ and a . For an atomic electron undergoing spontaneous excitation from, e.g., the 1s state of an atom with atomic number Z to a higher energy state, bound or free, the order of magnitude of \dot{P}_1 is $\approx g_e^2 10^{-23}/Z^2 \text{sec}^{-1}$. For, e.g., a proton in an outer shell of a nucleus of mass number A it is $\approx g_p^2 10^{-32} A^{2/3} \text{sec}^{-1}$. With such rates, the 1s electrons in 10^{24} such atoms would be expected to provide $\approx g_e^2 10/Z^2$ photon pulses each second while each spontaneously excited proton in 10^{24} such nuclei would be expected to provide $\approx g_p^2 3A^{2/3}$ gammas each year. This large difference in rates explains why we are able to obtain good experimental limits on the electron's coupling constant g_e in section 6, but not on the neutron's coupling constant g_n in section 5.

Although we shall only apply Eq. (3.2) in our data analysis, for completeness we include Eqs. (3.4), (3.5) below which could be applied if more accurate data becomes available. We note again that if g_α is mass-proportional (i.e., if for protons $g_p = 1$, then for neutrons $g_n \approx 1.001$ and for electrons $g_e \approx .00054$), then $\mathbf{R} \equiv 0$, and therefore (3.2) vanishes. We should then need \dot{P} to order a^{-4} :

$$\dot{P}_2 = \frac{\lambda}{16a^4} [|\langle \phi | S | \psi \rangle|^2 + 2 \sum_{m=1}^3 \sum_{n=1}^3 |\langle \phi | S^{mn} | \psi \rangle|^2] \quad (3.4)$$

where $S^{mn} \equiv \sum_{j=1}^N g_{\alpha(j)} (\mathbf{R}_j)^m (\mathbf{R}_j)^n$, $S \equiv \sum_{n=1}^3 S^{nn}$. If \mathbf{R}_j corresponds to a charged particle, its matrix elements here describe electric monopole or quadrupole transitions. These matrix elements are smaller than the corresponding matrix elements in Eq.(3.2) by the factor (size of bound state/ a)².

Fu¹⁰ has considered the spontaneous electromagnetic radiation of a free charged particle in CSL, obtaining the probability/sec/energy of radiating a photon of energy $E = \hbar k$:

$$\frac{d\dot{P}(E)}{dE} = g_\alpha^2 \frac{\lambda}{4\pi^2} \frac{e^2}{\hbar c} \left(\frac{\hbar/M_\alpha c}{a} \right)^2 \frac{1}{E} \approx \frac{3 \cdot 10^{-31}}{E \text{ in keV}} \text{ counts/sec/keV} \quad (3.5a, b)$$

(the infrared divergence is treated as usual). Eq.(3.5b) gives the rate for an electron with $g_e = 1$. This radiation rate (3.5) for free particles is smaller than the excitation rate (3.2) for bound particles by the factor $e^2/\hbar c \approx 1/137$ and by the replacement of (bound state size/ a)² by (Compton wavelength/ a)². We

note, with mass-proportionality, that the spontaneous radiation rate for free electrons is the same as for free protons, on account of the factor $(g_\alpha/M_\alpha)^2$ in (3.5a).

This completes our collection of equations giving CSL spontaneous excitation rates.

We now consider the spontaneous excitation of valence nucleons in a Germanium nucleus and the spontaneous ionization of 1s electrons in a Germanium atom, using data from what are, at present, the lowest noise relevant experiments.

4. Experiments.

The data used in the analysis¹⁸ of the atomic excitation comes from a small (253 g) p-type coaxial HPGe crystal, “COSME”, built specifically for a Dark Matter search¹⁹. It features, at only 1.6 keV, the (so-far) lowest energy threshold of any detector dedicated to such searches, and has as well an excellent resolution of 0.43 keV (FWHM) at 10.3 keV. Special low-radioactivity measures were taken, such as mounting the detector on a specially-designed electroformed copper cryostat, shielding of electronic components close to the crystal with 450-yr-old lead, and use of a 2000-yr-old roman lead layer in the innermost part of the shielding. Additional photon, neutron and vibrational shielding were used.

The detector set-up was installed in the Canfranc-1 underground laboratory in the Spanish Pyrenees, at a depth of 675 meters of water equivalent. The microphonic component characteristic of very low-threshold detectors, extending up to ≈ 15 keV, was filtered-out using specially-developed techniques²⁰. While the low-energy background level was slightly higher than that in the “TWIN” detectors¹³, the improved resolution—typically inversely proportional to the mass of the crystal—allows one to impose more stringent limits on a sharply defined signal that might be buried in an otherwise featureless background, as is the case for the emitted radiation in CSL.

At this point a remark is in order: unfortunately the TWIN data used to extract CSL limits on g_e in reference 12 belonged to a preliminary set coming from un-amplified digitized pulses. Later comparison with the spectrum collected with a multichannel analyzer showed that this earlier data was corrupted at energies below ≈ 200 keV, i.e., a large fraction of events were not recorded. This faulty set was not used in other TWIN results, namely for double-beta decay¹³, Dark Matter²¹ or electron half-life²². In reference 12, only one data point was used, the (erroneous) rate .049 counts/keV/kg/day at 11 keV (where the highest predicted value of spontaneous radiation occurs) to set an upper limit on g_e . In the present work, it is necessary to perform a χ^2 analysis of a fit to the whole predicted shape of the spontaneous radiation, in order to obtain a limit comparable to the (erroneous) limit of reference 12.

The data used in the analysis of the nuclear excitation comes from the “Rico Grande” crystals, which are part of the IGEX ensemble of large enriched germanium detectors²³, dedicated to searching for neutrinoless double-beta decay. The two detectors from which these data were extracted have a fiducial mass of ≈ 2 kg each and are enriched to 86% Ge_{76} and 14% Ge_{74} . As a result, the prevailing source of background in the energy neighborhood of interest here is this two-neutrino double-beta decay from Ge_{76} which, however, cuts off at an energy below the region employed in the analysis in section 5. At the time of collection of the present data, the Ricos were operated in the Homestake mine in similar conditions to TWIN.

5. Constraint on g_n/g_p ?

We now apply Eq. (3.2) to the spontaneous excitation rate of a single proton or neutron in a Ge nucleus.

The Jparity for the ground state of Ge is 0+ for the even-even nuclides Ge_{70} (20.6%), Ge_{72} (27.4%), Ge_{74} (36.7%) and Ge_{76} (7.7%), and it is 9/2+ for Ge_{73} (7.7%). Spontaneous excitation is predicted (the matrix elements are nonvanishing) for a transition from the ground state to 1- states in the case of the even-even nuclides and to 11/2-, 9/2- or 7/2- states in Ge_{73} . From the point of view of the shell model, among other possibilities, a proton or neutron can be excited from its ground state valence level to a higher energy state.

The return of a proton from the excited state to ground could be direct, via an electric dipole transition. It could also proceed indirectly, through intermediate states or internal conversion and, in the case of a

neutron it must proceed indirectly, through magnetic transitions. In any case, the lifetimes are in most cases so short that a photon pulse would rapidly appear at the energy difference of the two states. Therefore, by looking at the data for a signature peak of instrumental resolution width at the expected energy, one may hope to observe the radiation resulting from these spontaneous transitions or at least get an upper limit on their rate.

Not only does the matrix element of \mathbf{R}_i for the excited particle not vanish, but the matrix element of \mathbf{R}_i for the other particles also does not vanish due to their dependence on the center of mass operator. We can relate the matrix element of the protons to that of the neutrons by using $\Sigma_j M_j \mathbf{R}_j \equiv 0$, which implies that

$$\Sigma_{i=1}^{A-Z} \mathbf{R}_{ni} = -(M_p/M_n) \Sigma_{i=1}^Z \mathbf{R}_{pi} \quad (5.1a)$$

(we neglect the electron contribution of $o(M_e/(M_n))$ so, setting $M_p/M_n = 1$, the matrix element in (3.2) is

$$\langle \phi | g_p \Sigma_{i=1}^Z \mathbf{R}_{pi} + g_n \Sigma_{i=1}^{A-Z} \mathbf{R}_{ni} | \psi \rangle = [1 - g_n] \langle \phi | \Sigma_{i=1}^Z \mathbf{R}_{pi} | \psi \rangle \quad (5.1b)$$

(remembering $g_p \equiv 1$). Thus, from Eqs. (3.2) and (5.1), the excitation rate Γ in sec^{-1} of e.g., one of the four valence protons from the ground state $|\psi\rangle$ to one of the 12 degenerate excited states $|\phi\rangle$ (any of the four protons may be excited, and the 1- state of this proton plus its subshell partner can have three possible orientations) can be expressed purely in terms of proton matrix elements:

$$\begin{aligned} \Gamma &= \frac{\lambda}{2a^2} [1 - g_n]^2 12 |\langle \phi | \Sigma_{i=1}^Z \mathbf{R}_{pi} | \psi \rangle|^2 \\ &= \frac{\lambda}{2a^2} [1 - g_n]^2 12 \Sigma_m \frac{4\pi}{3} |\langle \phi | \Sigma_{i=1}^{i=Z} R_{pi} Y_{1,m}(\theta_i, \phi_i) | \psi \rangle|^2 \end{aligned} \quad (5.2a, b)$$

A similar expression may be written for the excitation rate of a neutron totally in terms of neutron matrix elements.

It is worth remarking that the expression for the lifetime $\tau(\phi \rightarrow \psi)$ of one of the excited state $|\phi\rangle$ to decay by an electric dipole transition to the ground state $|\psi\rangle$ can be written in terms of the same matrix elements as appear in (5.2)²⁴:

$$\frac{1}{\tau(\phi \rightarrow \psi)} = \frac{16\pi c}{9} \left(\frac{E}{\hbar c} \right)^3 \left(\frac{e^2}{\hbar c} \right) \Sigma_m |\langle \phi | \Sigma_{i=1}^{i=Z} R_{pi} Y_{1,m}(\theta_i, \phi_i) | \psi \rangle|^2 \quad (5.3)$$

where E is the energy difference of the two states. Thus we may express Γ in terms of this lifetime:

$$\Gamma = \frac{\lambda}{2a^2} [1 - g_n]^2 \frac{9}{c(e^2/\hbar c)} \left(\frac{\hbar c}{E} \right)^3 \frac{1}{\tau(\phi \rightarrow \psi)} \quad (5.4)$$

Eq. (5.4) would be useful if we have the experimental lifetimes and branching ratios of the state ϕ . Unfortunately, in this case we do not, so we are forced to estimate the matrix element in (5.2b).

In this paper we shall approximate the matrix element by using the same “very rough estimate” employed by Blatt and Weisskopf²⁵ for calculating the lifetime τ . They assume the radial wavefunction of the proton in both states ϕ and ψ is $\Theta(R_0 - R)[3/R_0^3]^{1/2}$ where Θ is the step function and $R_0 = 1.4 \times 10^{-13} A^{1/3}$ is the nuclear radius. As they point out, the actual radial integral is expected to be “somewhat smaller” since radial wavefunctions oscillate: say, β times smaller. We obtain from (5.2b) the result

$$\Gamma = \frac{\lambda}{a^2} [1 - g_n]^2 \beta^2 R_0^2 \quad (5.5)$$

(a numerical factor 9/8 has been replace by 1).

The expected total count \mathcal{C} for an experimental run of D kg-days from a transition due to a nuclide which comprises the fraction X of the 8.3×10^{24} atoms/kg in common Ge is found from (5.5), with the GRW parameters, to be

$$\mathcal{C} = \frac{\beta^2}{4} [1 - g_n]^2 X D \quad (5.6)$$

For example, consider a transition in Ge_{74} to the 2165 keV 1- state. We shall use the data from the Rico Grande experiment²², similar to COSME but with $D = 1.135 \text{ kg-yrs} = 414.3 \text{ kg-days}$ (COSME's $D = 85.24 \text{ kg-days}$) with $X = .14$ (COSME's $X = .37$). Denoting by $\mathcal{C}_{\text{expt}}$ the upper limit on the number of observed counts, we obtain

$$1 + .26 \frac{\mathcal{C}_{\text{expt}}^{\frac{1}{2}}}{\beta} \geq g_n \geq 1 - .26 \frac{\mathcal{C}_{\text{expt}}^{\frac{1}{2}}}{\beta} \quad (5.7)$$

For this experiment, the upper limit (obtained from the counts under a Chi^2 fit to a background quadratic plus the expected experimental resolution shape centered on 2165 keV) is $\mathcal{C}_{\text{expt}} = .89$ counts at the 68% confidence level (3.9 counts at the 95% confidence level). For $\beta \approx .3$ one obtains $1.8 \geq g_n \geq .2$. However, our choice of β is just hypothetical, as we have not made the effort to seriously evaluate the matrix element^{26,27}. The points to be made are that the range of g_n is not so far from $g_p = 1$ and that the various numbers involved in calculating (5.7) tantalizingly contrive to be on the edge of showing mass proportionality. Indeed, this would more easily be shown with a larger value of λ/a^2 than the GRW value: for instance, with $\lambda/a^2 = 100\lambda/a_{\text{GRW}}^2$, the above inequality becomes $1.1 \geq g_n \geq .9$. But, with the GRW parameters, it would require a long counting time and a proper calculation of matrix elements before one might say that $g_n/g_p \approx 1$.

6. Constraint on g_e/g_p .

Here we apply Eq. (3.2) to calculate the spontaneous ionization rate of the 1s electrons in a Ge atom.

If a 1s electron is spontaneously ionized, the remaining electrons in the atom rapidly cascade into the (singly ionized) ground state, emitting a photon pulse of 11.1 keV (the ionization energy of a 1s electron). The ionized electron also deposits its kinetic energy in the Ge sample, which augments the energy of the pulse. Thus the signature of these events is a distribution of photon pulses of energy $E > 11.1 \text{ keV}$.

In reference 12, a Hartree calculation of the matrix element in (3.2) for the electrons was numerically performed, where $|\psi\rangle$ is the ground state of Ge and $|\phi\rangle$ is a state where a 1s electron is ionized. The result of the calculation may be expressed as a function $C(E)$ which gives the expected pulse counting rate if GRW parameters are assumed and if the electron is totally responsible for collapse (i.e., $g_e = 1$, $g_n = g_p = 0$). $C(E)$ is zero for $E < 11.1 \text{ keV}$, abruptly rises to 5370 counts/keV/kg/day at $E = 11.1 \text{ keV}$, and decays in roughly exponential fashion, with the value ≈ 4000 counts/keV/kg/day at $E = 12 \text{ keV}$, and ≈ 1500 counts/keV/kg/day at $E = 16 \text{ keV}$.

If we assume that $g_n = g_p = 1$ (as we shall hereafter do) then, as in the preceding section's Eq. (5.1), we can express the matrix element for the nucleons as $-M_e/M_p$ times the matrix element for the electrons. Putting this into Eq. (3.2), the resulting rate Γ in counts/sec/kg/day may then be written as

$$\Gamma = \frac{(\lambda/a^2)}{(\lambda/a^2)_{\text{GRW}}} \left[g_e - \frac{M_e}{M_p} \right]^2 C(E) \quad (6.1)$$

We note that the rate in (6.1) vanishes if there is mass proportionality ($g_e/g_p = M_e/M_p$).

Figure 1 shows a graph of the counts/.1keV/85.24kg-day from COSME in the energy range 5 to 17 keV. A recent paper²¹ describes a search with the same apparatus in a similar energy range. These authors were looking at the TWIN data for a signature 11.1 keV peak resulting from a hypothesized violation of charge conservation (in which a K-shell electron decays to neutrals, and the other electrons in the atom readjust). They fit the data to three x-ray peaks (Cu, Zn and Ga) known to result from cosmogenic excitation of the Ge isotopes in the sample under observation, together with a background quadratic polynomial plus the hypothesized process, and they look at the difference between the fit and the data at the 68% and 90% confidence levels.

We employ here the same procedure. Fig. 1 (solid line) shows the best fit to the data by this method, without the hypothesized process, a multiple of $C(E)$. Superimposed upon this fit is a graph of $10^{-3}C(E)$ (dash-dotted line), folded in with the detector resolution shape (a gaussian of standard deviation .18 keV). It is clear that this is by no means a good fit to the data, so the coefficient of $C(E)$ in Eq. (6.1) is considerably smaller than 10^{-3} .

Also shown in Fig. 1 is the fit with the hypothesized process at $4.2 \times 10^{-5}C(E)$ (dotted line) which corresponds to the 68% confidence level. Not shown is a similar shaped curve at $9.7 \times 10^{-5}C(E)$ which corresponds to the 95% confidence level.

If we assume that the parameters λ and a are the same as those given by GRW, then we conclude from Eq.(6.1) that, at the 68% confidence level, $[g_e/g_p - M_e/M_p]^2 C(E) \leq 4.2 \times 10^{-5}C(E)$ or

$$0 \leq \frac{g_e}{g_p} \leq 13 \frac{M_e}{M_p} \quad (6.2)$$

Thus, according to CSL with the GRW parameters, nucleons are mostly responsible for collapse.

It is worth noting that it would take an improvement in the experimental limit by e.g., a factor of 1/300, which results in $.3M_e/M_p \leq g_e/g_p \leq 1.7M_e/M_p$, to suggest that $g_e/g_p = 0$ may be ruled out. However, it should also be noted that we need not be wedded to the GRW parameters. Thus an increase of λ/a^2 by a factor of 300 or more would have the same effect. On the other hand it would take a decrease in λ/a^2 by a factor of 4×10^{-5} or more for the limit obtained in this experiment not to suggest that nucleons collapse more rapidly than electrons.

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Figure Captions

Figure. 1 A graph of the COSME data in the region 5-17 keV is shown along with the best fit to the three known X-ray peaks plus a quadratic polynomial background (solid curve). Two additional curves are shown, corresponding to predicted rates folded in with the experimental resolution (a gaussian of width .18keV). The dash-dotted curve corresponds to $10^{-3}C(E)$ and the dotted curve to the 68% confidence level value of $4.2 \times 10^{-5}C(E)$.